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## Lista Nr 3

# Granica funkcji w punkcie

### 3.1 Obliczanie granic funkcji

#### 3.1.1 Obliczyć podane granice funkcji:

- $\lim_{x \rightarrow 2} \frac{x^3 - 8}{x - 2};$
- $\lim_{x \rightarrow 3} \frac{27 - x^3}{x - 3};$
- $\lim_{x \rightarrow 3} \frac{x^2 - 4x + 3}{2x - 6};$
- $\lim_{x \rightarrow 4} \frac{x^2 - 2x - 4}{x^2 - 9x + 20};$
- $\lim_{x \rightarrow 0} \frac{\sqrt[3]{1 + 3x} - 1}{x};$
- $\lim_{x \rightarrow 0} \frac{\sqrt{x^2 + 1} - \sqrt{x + 1}}{1 - \sqrt{x + 1}};$
- $\lim_{x \rightarrow 0} \frac{\sqrt{x^2 + 1} - 1}{\sqrt{x^2 + 25} - 5};$
- $\lim_{x \rightarrow 0} \frac{\sqrt{x^2 + 1} - 1}{\sqrt{x^2 + 16} - 4};$
- $\lim_{x \rightarrow \infty} (\sqrt{x^2 + 1} - \sqrt{x^2 - 1});$
- $\lim_{x \rightarrow 5} \frac{\sqrt{x - 1} - 2}{x - 5};$
- $\lim_{x \rightarrow \infty} \left( \frac{x^3}{x^2 + 1} - x \right);$
- $\lim_{x \rightarrow 1} \frac{x^2 - x}{x^2 - 1};$
- $\lim_{x \rightarrow -2} \frac{x^2 + 3x + 2}{x^2 + 5x + 6};$
- $\lim_{x \rightarrow 1} \frac{x^3 - 2x^2 + x}{x^2 - 2x - 3};$
- $\lim_{x \rightarrow 1/2} \frac{2x^2 + 5x - 3}{4x^3 - 4x^2 + x};$
- $\lim_{x \rightarrow \infty} \frac{x^3}{x^2 - x + 2};$
- $\lim_{x \rightarrow \infty} \frac{\sqrt{4x^2 - x}}{x + 3};$
- $\lim_{x \rightarrow \infty} \frac{x^2 - x + 4}{\sqrt{x^3 + 2}};$
- $\lim_{x \rightarrow \infty} \frac{2}{5x};$
- $\lim_{x \rightarrow \infty} \frac{2x^2 + 2}{x - 3};$
- $\lim_{x \rightarrow \infty} \frac{6x^2}{2x^2 - 5};$
- $\lim_{x \rightarrow \infty} (\sqrt{x^2 + 2x} - \sqrt{x^2 - 2x});$
- $\lim_{x \rightarrow \infty} (\sqrt{x^2 - x + 1} - x);$
- $\lim_{x \rightarrow -\infty} (\sqrt{1 - x} + x).$

#### 3.1.2 Opierając się na I i II granice podstawowe obliczyć granice funkcji:

- $\lim_{x \rightarrow \infty} \left( 1 + \frac{2}{3x} \right)^{-x};$
- $\lim_{x \rightarrow \infty} \left( \frac{x+1}{x-3} \right)^{2x-1};$
- $\lim_{x \rightarrow \infty} \left( \frac{x}{1+x} \right)^x;$
- $\lim_{x \rightarrow 0} \frac{\ln(1+2x)}{x};$
- $\lim_{x \rightarrow \infty} \left( \frac{x+1}{x-2} \right)^{2x-1};$
- $\lim_{x \rightarrow 0} (1 + \sin x)^{2x};$
- $\lim_{x \rightarrow 0} \frac{\sin 7x}{\sin 5x};$
- $\lim_{x \rightarrow 0} \frac{e^x - 1}{\sin 2x};$
- $\lim_{x \rightarrow 0} \frac{3^x - 2^x}{x};$
- $\lim_{x \rightarrow \infty} \left( \frac{3x+5}{3x+7} \right)^{x+1};$
- $\lim_{x \rightarrow \infty} \left( \frac{x^2+5}{x^2-7} \right)^{x^2};$
- $\lim_{x \rightarrow \infty} \left( 1 + \frac{1}{x^2} \right)^{2x-1};$
- $\lim_{x \rightarrow 0} \frac{(1-2x^3)}{x^3};$
- $\lim_{x \rightarrow 0} \sin 3x \operatorname{Ctg} 5x;$
- $\lim_{x \rightarrow 0^+} \frac{\operatorname{tg} \sqrt[3]{x}}{\sqrt{x}};$

$$\begin{array}{lll}
16. \lim_{x \rightarrow \frac{\pi}{2}} (1 + \cos x)^{\frac{1}{2x-\pi}}; & 17. \lim_{x \rightarrow \infty} (e^x \operatorname{th}(e^{-x})); & 18. \lim_{x \rightarrow 0} \frac{\arcsin 3x}{\arcsin 2x}; \\
19. \lim_{x \rightarrow 0} \frac{7^x - 5^x}{3^x - 2^x}; & 20. \lim_{x \rightarrow 1} \frac{5 \cdot 3^x - 3 \cdot 5^x}{2 \cdot 7^x - 7 \cdot 2^x}; & 21. \lim_{x \rightarrow 0} \frac{\ln(1+x)}{e^x - 1}; \\
22. \lim_{x \rightarrow \frac{\pi}{2}} \frac{\operatorname{tg} 3x}{\operatorname{tg} 5x}; & 23. \lim_{x \rightarrow \pi} \frac{\sin 2x}{\sin 7x}; & 24. \lim_{x \rightarrow \frac{\pi}{2}} \frac{\ln(1+\cos)}{\ln(1+\cos 3x)}; \\
25. \lim_{x \rightarrow 0} \frac{\operatorname{sh} x}{\operatorname{sh} \sqrt{x}}; & 26. \lim_{x \rightarrow 0} \frac{\operatorname{arctg} x}{\operatorname{tg} x}; & 27. \lim_{x \rightarrow 0} \frac{\ln \cos x}{x^2};
\end{array}$$

### 3.1.3 Obliczyć granice jednostronne:

$$\begin{array}{lll}
1. \lim_{x \rightarrow 0^+} \frac{x}{|x|}; & 2. \lim_{x \rightarrow 1^-} \frac{|x^4 - 1|}{x - 1}; & 3. \lim_{x \rightarrow 3^+} [-x]; \\
4. \lim_{x \rightarrow 2\pi^-} \operatorname{sgn}(\sin x); & 5. \lim_{x \rightarrow 0^+} \operatorname{arctg} \frac{1}{x}; & 6. \lim_{x \rightarrow 0^-} e^{\frac{1}{x}}.
\end{array}$$

### 3.1.4 Zbadać, czy podane proste są asymptotami pionowymi wskazanych funkcji:

$$\begin{array}{lll}
1. f(x) = \frac{\sin^2 x}{x}, x = 0; & 2. f(x) = \ln(4-x), x = 4; & 3. f(x) = \frac{e^{-x} - 1}{e^x - 1}, x = 0; \\
4. f(x) = e^{\frac{1}{x}}, x = 0; & 5. f(x) = \frac{x^3}{x-1}, x = 1; & 6. f(x) = \frac{x-2}{\sqrt{4-x^2}}, x = \pm 2.
\end{array}$$

### 3.1.5 Znaleźć asymptoty pionowe i ukośne podanych funkcji:

$$\begin{array}{lll}
1. f(x) = \frac{x}{1-x}; & 2. f(x) = x - 2\sqrt{x}; & 3. f(x) = \sqrt{x^2 - 1}; \\
4. f(x) = \frac{x^3 + 8}{x^2 - 4}; & 5. f(x) = \frac{\sin x}{x^2}; & 6. f(x) = \cos \frac{1}{x}; \\
7. f(x) = e^{-\frac{1}{x^2}}; & 8. f(x) = \frac{x^2 + 2x}{x+1}; & 9. f(x) = \frac{x}{\operatorname{arctg} x}.
\end{array}$$